|  |  |
| --- | --- |
| **Inputs** | **Outputs** |
| **Simple Cases (**; ) | |
| In principal frame:  In inertial frame: | In principal frame:  In inertial frame: |
| In principal frame:  In inertial frame: | In principal frame (after  In inertial frame: |
| In principal frame:  In inertial frame: | In principal frame (after  In inertial frame: |
| In principal frame:  In inertial frame: | If the angular momentum vector should be the same at the beginning of simulation and at the end because angular moment remains constant.  In principal frame (after  In inertial frame: |
| In principal frame:  In inertial frame: | In principal frame ():  In inertial frame: |
| In principal frame:  In inertial frame: | In principal frame ():  In inertial frame: |
| In principal frame:  In inertial frame: | In principal frame ():  In inertial frame: |
| **Iso-inertial (**) | |
| In principal frame: | In principal frame ( |
| **Axisymmetric (**) | |
| In principal frame: | In principal frame (    Figure 10: Prolate body cone half-angle and space cone half-angle over time.    Figure 11: Prolate angular velocity components over time. |
| In principal frame: | In principal frame (    Figure 12: Oblate body cone half-angle and space cone half-angle over time.    Figure 13: Oblate angular velocity components over time. |
| **Tri-inertial (**) | |
| In principal frame: | Figure 14: Energy and momentum ellipsoids intersection for major axis case with angular velocity vector overlaid. Animated vector can be found in the Project 1 live script in the linked GitHub repository. |
| In principal frame: | Check that the spin is stable. Plot the intersection of energy and momentum ellipsoids, the angular velocity vector should trace the intersection over time.  The energy and momentum ellipsoids for these initial conditions look like:    Figure 15: Energy and momentum ellipsoids intersection for minor axis case with angular velocity vector overlaid. Animated vector can be found in the Project 1 live script in the linked GitHub repository. |
| In principal frame: | Figure 16: Energy and momentum ellipsoids intersection for intermediate axis case with angular velocity vector overlaid. Animated vector can be found in the Project 1 live script in the linked GitHub repository. |
| **General Case** | |
| In the body frame:  In inertial frame: | In the inertial frame (: |

## Surrogate Phone Satellite Experimental Data and Analysis

An iPhone 13 Pro was used to experimentally test the Simulink model. For all test cases, data was obtained by starting a sensor log on the phone to record its motion. The phone was tossed as precisely as possible for the acquired data sets. The data was then parsed to plot only relevant periods when the phone was subject to pure rotational kinematics. The first test case was pure spin about the phone’s major axis. Reference the figures below.

A graph with colorful lines

Description automatically generated A graph of different colored lines

Description automatically generated

|  |  |
| --- | --- |
| Figure 17: Measured and predicted angular velocity components overlayed for major axis spin of phone satellite, plotted over time. | Figure 18: Measured and predicted Euler Angles overlayed for major axis spin of phone satellite, plotted over time. |

By observation, the model’s prediction of the angular velocity over time is highly accurate. However, discrepancies can be observed in the plot of Euler Angles over time. The phone’s on-board gyroscope uses a left-handed coordinate system due to compass azimuth convention. The data had to be adjusted manually to correct it. Additionally, a phase shift can be seen between the predicted and actual angle and may be explained by sensor error of miscalibration. The model accurately predicts the response of the measured Euler Angles as seen by the shape of the curves despite being out of phase.

The rotational energy of the system was plotted over time in Figure 19 below.

A graph of energy and time

Description automatically generated

Figure 19: Actual and theoretical rotational energy for major axis spin of phone satellite plotted over time.

By observation, the model shows accuracy in predicting the actual rotational energy of the phone satellite. Spinning about the major axis for a tri-inertial object is its theoretical lowest energy state, and therefore, given the same initial conditions about a different axis, it is expected that the rotational energy of the system spinning about any other axis be greater than that of the major axis. However, due to human inconsistency, this was not feasibly produceable.

The next test case was pure spin about minor axis. Reference the figures below.

A graph with colored lines

Description automatically generated A graph of colored lines

Description automatically generated

|  |  |
| --- | --- |
| Figure 20: Measured and predicted angular velocity components overlayed for minor axis spin of phone satellite, plotted over time. | Figure 21: Measured and predicted Euler Angles overlayed for minor axis spin of phone satellite, plotted over time. |

As in the first case, similar simulation-model accuracies and discrepancies may be observed. For the system’s rotational energy, reference the figure below.

A graph of energy and theoretical energy

Description automatically generated

Figure 22: Actual and theoretical rotational energy for minor axis spin of phone satellite, plotted over time.

Then, the intermediate axis test case. Reference the figures below.

A graph of different colored lines

Description automatically generated A graph of different colored lines

Description automatically generated

|  |  |
| --- | --- |
| Figure 23: Measured and predicted angular velocity components overlayed for intermediate axis spin of phone satellite, plotted over time. | Figure 24: Measured and predicted Euler Angles overlayed for intermediate axis spin of phone satellite, plotted over time. |

The rotational energy of the phone satellite rotating about its intermediate axis is shown below in Figure 25.

A graph of energy and time

Description automatically generated

Figure 25: Actual and theoretical rotational energy for intermediate axis spin of phone satellite, plotted over time.